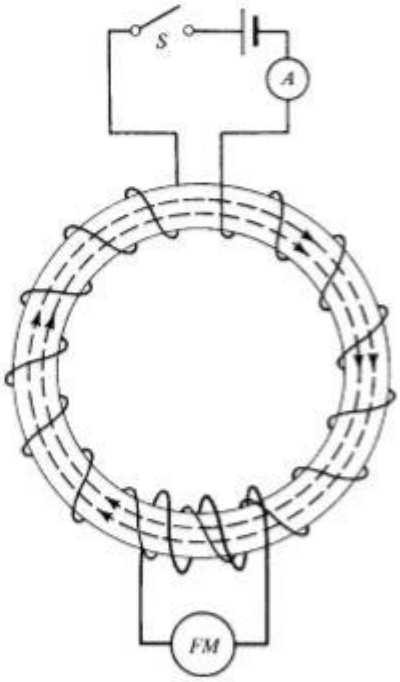

Fig. 1.10 Circuit for magnetization of a ring. Dashed lines indicate flux.

region within the coil. This arrangement has the advantage that the material of the ring becomes magnetized without the formation of poles, which simplifies the interpretation of the measurement. Another winding, called the secondary winding or search coil, is placed on all or a part of the ring and connected to an electronic integrator or fluxmeter. Some practical aspects of this measurement are discussed in Chapter 2.

Let us start with the case where the ring contains nothing but empty space. If the switch S is closed, a current i is established in the primary, producing a field of H oersteds, or maxwells/cm^2, within the ring. If the cross-sectional area of the ring is A cm^2, then the total number of lines of force in the ring is $HA = \Phi$ maxwells, which is called the *magnetic flux*. (It follows that H may be referred to as a flux density.) The change in flux $\Delta\Phi$ through the search coil, from 0 to Φ, induces an electromotive force (emf) in the search coil according to Faraday's law:

$$E = -10^{-8} n \left(\frac{d\Phi}{dt} \right) \quad \text{or} \quad \int E\, dt = -10^{-8} n\, \Delta\Phi,$$

where n is the number of turns in the secondary winding, t is time in seconds, and E is in volts.

The (calibrated) output of the voltage integrator $\int E\, dt$ is a measure of $\Delta\Phi$, which in this case is simply Φ. When the ring contains empty space, it is found that Φ_{observed}, obtained from the integrator reading, is exactly equal to Φ_{current}, which is the flux produced by the current in the primary winding, i.e., the product A and H calculated from Equation 1.13.

However, if there is any material substance in the ring, $\Phi_{observed}$ is found to differ from $\Phi_{current}$. This means that the substance in the ring has added to, or subtracted from, the number of lines of force due to the field *H*. The relative magnitudes of these two quantities, $\Phi_{observed}$ and $\Phi_{current}$, enable us to classify all substances according to the kind of magnetism they exhibit:

$\Phi_{observed} < \Phi_{current}$,	diamagnetic (i.e., Cu, He)
$\Phi_{observed} > \Phi_{current}$,	paramagnetic (i.e., Na, Al)
	or antiferromagnetic (i.e., MnO, FeO)
$\Phi_{observed} \gg \Phi_{current}$,	ferromagnetic (i.e., Fe, Co, Ni)
	or ferrimagnetic (i.e., Fe_3O_4)

Paramagnetic and antiferromagnetic substances can be distinguished from one another by magnetic measurement only if the measurements extend over a range of temperature. The same is true of ferromagnetic and ferrimagnetic substances.

All substances are magnetic to some extent. However, examples of the first three types listed above are so feebly magnetic that they are usually called "nonmagnetic," both by the layman and by the engineer or scientist. The observed flux in a typical paramagnetic, for example, is only about 0.02% greater than the flux due to the current. The experimental method outlined above is not capable of accurately measuring such small differences, and entirely different methods have to be used. In ferromagnetic and ferrimagnetic materials, on the other hand, the observed flux may be thousands of times larger than the flux due to the current.

We can formally understand how the material of the ring causes a change in flux if we consider the fields which actually exist inside the ring. Imagine a very thin, transverse cavity cut out of the material of the ring, as shown in Fig. 1.11. Then *H* lines/cm^2 cross this gap, due to the current in the magnetizing winding, in accordance with Equation 1.13. This flux density is the same whether or not there is any material in the ring. In addition, the applied field *H*, acting from left to right, magnetizes the material, and north and south poles are produced on the surface of the cavity, just as poles are produced on the ends of a magnetized bar. If the material is ferromagnetic, the north poles will be on the left-hand surface and south poles on the right. If the intensity of magnetization is *M*, then each square centimeter of the surface of the cavity has a pole strength of *M*, and $4\pi M$ lines issue from it. These are sometimes called *lines of magnetization*. They add to the *lines of force* due to the applied field *H*, and the combined group of lines crossing the gap are called *lines of magnetic flux* or

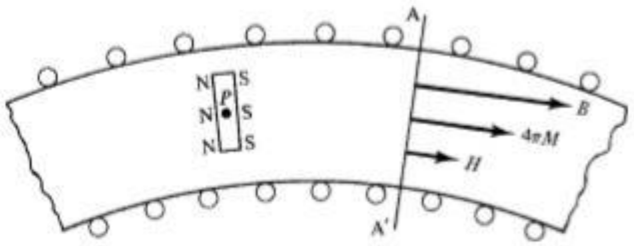

Fig. 1.11 Transverse cavity in a portion of a Rowland ring.

lines of induction. The total number of lines per cm^2 is called the *magnetic flux density* or the *induction B*. Therefore,

$$B = H + 4\pi M. \tag{1.15}$$

The word "induction" is a relic from an earlier age: if an unmagnetized piece of iron were brought near a magnet, then magnetic poles were said to be "induced" in the iron, which was, in consequence, attracted to the magnet. Later the word took on the quantitative sense, defined above, of the total flux density in a material, denoted by B. Flux density is now the preferred term.

Because lines of B are always continuous, Equation 1.15 gives the value of B, not only in the gap, but also in the material on either side of the gap and throughout the ring. Although B, H, and M are vectors, they are usually parallel, so that Equation 1.15 is normally written in scalar form. These are vectors indicated at the right of Fig. 1.11, for a hypothetical case where B is about three times H. They indicate the values of B, H, and $4\pi M$ at the section AA' or at any other section of the ring.

Although B, H, and M must necessarily have the same units (lines or maxwells/cm^2), different names are given to these quantities. A maxwell per cm^2 is customarily called a *gauss* (G),[4] when it refers to B, and an *oersted* when it refers to H. However, since in free space or (for practical purposes) in air, $M = 0$ and therefore $B = H$, it is not uncommon to see H expressed in gauss. The units for magnetization raise further difficulties. As we have seen, the units for M are erg/Oe cm^3, commonly written emu/cm^3, but $4\pi M$, from Equation 1.15, must have units of maxwells/cm^2, which could with equal justification be called either gauss or oersteds. In this book when using cgs units we will write M in emu/cm^3, but $4\pi M$ in gauss, to emphasize that the latter forms a contribution to the total flux density B. Note that this discussion concerns only the names of these units (B, H, and $4\pi M$). There is no need for any numerical conversion of one to the other, as they are all numerically equal. It may also be noted that it is not usual to refer, as is done above, to H as a flux density and to HA as a flux, although there would seem to be no logical objection to these designations. Instead, most writers restrict the terms "flux density" and "flux" to B and BA, respectively.

Returning to the Rowland ring, we now see that $\Phi_{\text{observed}} = BA$, because the integrator measures the change in the total number of lines enclosed by the search coil. On the other hand, $\Phi_{\text{current}} = HA$. The difference between them is $4\pi MA$. The magnetization M is zero only for empty space. The magnetization, even for applied fields H of many thousands of oersteds, is very small and negative for diamagnetics, very small and positive for paramagnetics and antiferromagnetics, and large and positive for ferro- and ferrimagnetics. The negative value of M for diamagnetic materials means that south poles are produced on the left side of the gap in Fig. 1.11 and north poles on the right.

Workers in magnetic materials generally take the view that H is the "fundamental" magnetic field, which produces magnetization M in magnetic materials. The flux density B is a useful quantity primarily because changes in B generate voltages through Faraday's law.

The magnetic properties of a material are characterized not only by the magnitude and sign of M but also by the way in which M varies with H. The ratio of these two

[4]Carl Friedrich Gauss (1777–1855), German mathematician was renowned for his genius in mathematics. He also developed magnetostatic theory, devised a system of electrical and magnetic units, designed instruments for magnetic measurements, and investigated terrestrial magnetism.

quantities is called the *susceptibility* χ:

$$\chi = \frac{M}{H} \; \frac{\text{emu}}{\text{Oe} \cdot \text{cm}^3}. \tag{1.16}$$

Note that, since M has units $\text{A} \cdot \text{cm}^2/\text{cm}^3$, and H has units A/cm, χ is actually dimensionless. Since M is the magnetic moment per unit volume, χ also refers to unit volume and is sometimes called the *volume susceptibility* and given the symbol χ_v to emphasize this fact. Other susceptibilities can be defined, as follows:

$\chi_m = \chi_v/\rho$ = mass susceptibility (emu/Oe g), where ρ = density,
$\chi_A = \chi_v A$ = atomic susceptibility (emu/Oe g atom), where A = atomic weight,
$\chi_M = \chi_v M'$ = molar susceptibility (emu/Oe mol), where M' = molecular weight.

Typical curves of M vs H, called *magnetization curves*, are shown in Fig. 1.12 for various kinds of substances. Curves (a) and (b) refer to substances having volume susceptibilities of -2×10^{-6} and $+20 \times 10^{-6}$, respectively. These substances (dia-, para-, or antiferromagnetic) have linear M, H curves under normal circumstances and retain no magnetism when the field is removed. The behavior shown in curve (c), of a typical ferro- or ferrimagnetic, is quite different. The magnetization curve is nonlinear, so that χ varies with H and passes through a maximum value (about 40 for the curve shown). Two other phenomena appear:

1. *Saturation*. At large enough values of H, the magnetization M becomes constant at its saturation value of M_s.
2. *Hysteresis*, or irreversibility. After saturation, a decrease in H to zero does not reduce M to zero. Ferro- and ferrimagnetic materials can thus be made into permanent magnets. The word *hysteresis* is from a Greek word meaning "to lag behind," and is today applied to any phenomenon in which the effect lags behind the cause,

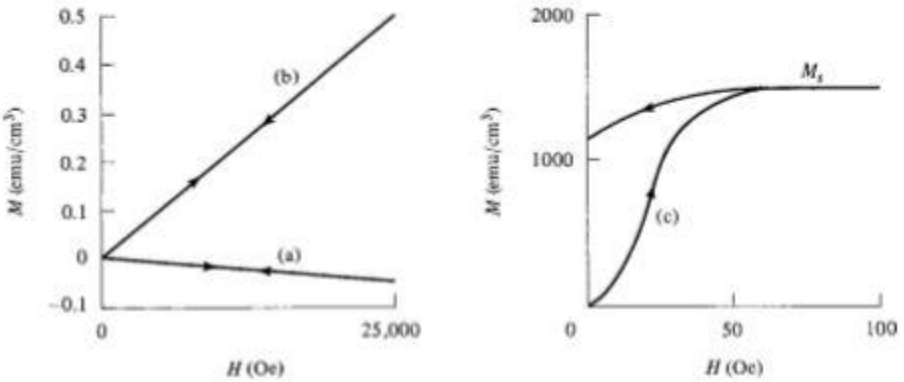

Fig. 1.12 Typical magnetization curves of (a) a diamagnetic; (b) a paramagnetic or antiferromagnetic; and (c) a ferromagnetic of ferrimagnetic.

leading to irreversible behavior. Its first use in science was by Ewing[5] in 1881, to describe the magnetic behavior of iron.

In practice, susceptibility is primarily measured and quoted only in connection with dia- and paramagnetic materials, where χ is independent of H (except possibly at very low temperatures and high fields). Since these materials are very weakly magnetic, susceptibility is of little engineering importance. Susceptibility is, however, important in the study and use of superconductors.

Engineers are usually concerned only with ferro- and ferrimagnetic materials and need to know the total flux density B produced by a given field. They therefore often find the B, H curve, also called a magnetization curve, more useful than the M, H curve. The ratio of B to H is called the *permeability* μ:

$$\mu = \frac{B}{H} \text{ (dimensionless).} \tag{1.17}$$

Since $B = H + 4\pi M$, we have

$$\frac{B}{H} = 1 + 4\pi\left(\frac{M}{H}\right),$$

$$\mu = 1 + 4\pi\chi. \tag{1.18}$$

Note that μ is not the slope dB/dH of the B, H curve, but rather the slope of a line from the origin to a particular point on the curve. Two special values are often quoted, the initial permeability μ_0 and the maximum permeability μ_{max}. These are illustrated in Fig. 1.13, which also shows the typical variation of μ with H for a ferro- or ferrimagnetic. If not otherwise specified, permeability μ is taken to be the maximum permeability μ_{max}. The local slope of the B, H curve dB/dH is called the *differential permeability*, and is sometimes

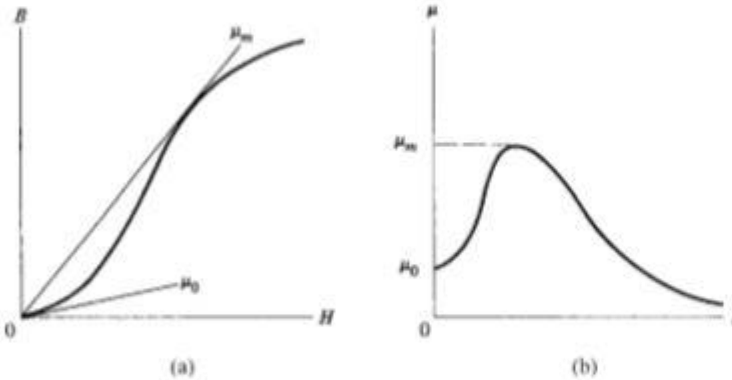

Fig. 1.13 (a) B vs H curve of a ferro- or ferrimagnetic, and (b) corresponding variation of μ with H.

[5] J. A. Ewing (1855–1935), British educator and engineer taught at Tokyo, Dundee, and Cambridge and did research on magnetism, steam engines, and metallurgy. During World War I, he organized the cryptography section of the British Admiralty. During his five-year tenure of a professorship at the University of Tokyo (1878–1883), he introduced his students to research on magnetism, and Japanese research in this field has flourished ever since.

used. Permeabilities are frequently quoted for soft magnetic materials, but they are mainly of qualitative significance, for two reasons:

1. Permeability varies greatly with the level of the applied field, and soft magnetic materials are almost never used at constant field.
2. Permeability is strongly structure-sensitive, and so depends on purity, heat treatment, deformation, etc.

We can now characterize the magnetic behavior of various kinds of substances by their corresponding values of χ and μ:

1. Empty space; $\chi = 0$, since there is no matter to magnetize, and $\mu = 1$.
2. Diamagnetic; χ is small and negative, and μ slightly less than 1.
3. Para- and antiferromagnetic; χ is small and positive, and μ slightly greater than 1.
4. Ferro- and ferrimagnetic; χ and μ are large and positive, and both are functions of H.

The permeability of air is about 1.000,000,37. The difference between this and the permeability of empty space is negligible, relative to the permeabilities of ferro- and ferrimagnetics, which typically have values of μ of several hundreds or thousands. We can therefore deal with these substances in air as though they existed in a vacuum. In particular, we can say that B equals H in air, with negligible error.

1.8 SI UNITS

The SI system of units uses the meter, kilogram, and second as its base units, plus the international electrical units, specifically the ampere. The concept of magnetic poles is generally ignored (although it need not be), and magnetization is regarded as arising from current loops.

The magnetic field at the center of a solenoid of length l, n turns, carrying current i, is given simply by

$$H = \frac{ni}{l} \, \frac{\text{ampere turns}}{\text{meter}}. \tag{1.19}$$

Since n turns each carrying current i are equivalent to a single turn carrying current ni, the unit of magnetic field is taken as A/m (amperes per meter). It has no simpler name. Note that the factor 4π does not appear in Equation 1.19. Since the factor 4π arises from solid geometry (it is the area of a sphere of unit radius), it cannot be eliminated, but it can be moved elsewhere in a system of units. This process (in the case of magnetic units) is called *rationalization*, and the SI units of magnetism are *rationalized mks units*. We will see shortly where the 4π reappears.

If a loop of wire of area A (m^2) is placed perpendicular to a magnetic field H (A/m), and the field is changed at a uniform rate $dH/dt =$ const., a voltage is generated in the loop according to Faraday's law:

$$E = -kA\left(\frac{dH}{dt}\right) \text{ volt}. \tag{1.20}$$

The negative sign means that the voltage would drive a current in the direction that would generate a field opposing the change in field. Examination of the dimensions of Equation 1.20 shows that the proportionality constant k has units

$$\frac{\mathrm{V \cdot sec}}{\mathrm{m^2 \cdot (A \cdot m^{-1})}} = \frac{\mathrm{V \cdot sec}}{\mathrm{A \cdot m}}.$$

Since

$$\frac{\mathrm{V}}{\mathrm{A \cdot sec^{-1}}}$$

is the unit of inductance, the henry (H), the units of k are usually given as H/m (henry per meter). The numerical value of k is $4\pi \times 10^{-7}\, H/m$; it is given the symbol μ_0 (or sometimes Γ), and has various names: the *permeability of free space*, the *permeability of vacuum*, the *magnetic constant*, or the *permeability constant*. This is where the factor 4π appears in rationalized units.

Equation 1.20 can alternatively be written

$$E = -A\left(\frac{dB}{dt}\right) \quad \text{or} \quad \int E dt = -A\,\Delta B. \tag{1.21}$$

Here B is the magnetic flux density (V sec/m^2). A line of magnetic flux in the SI system is called a weber (Wb = V sec), so flux density can also be expressed in Wb/m^2, which is given the special name of the *tesla* (T).[6]

In SI units, then, we have a magnetic field H defined from the ampere, and a magnetic flux density B, defined from the volt. The ratio between these two quantities (in empty space), B/H, is the magnetic constant μ_0.

A magnetic moment m is produced by a current i flowing around a loop of area A, and so has units $\mathrm{A \cdot m^2}$. Magnetic moment per unit volume $M = m/V$ then has units

$$\frac{\mathrm{A \cdot m^2}}{\mathrm{m^3}} = \mathrm{A \cdot m^{-1}},$$

the same as the units of magnetic field. Magnetization per unit mass becomes

$$\sigma = \frac{\mathrm{A \cdot m^2}}{\mathrm{w}}\left(\frac{\mathrm{A \cdot m^2}}{\mathrm{kg}} \text{ or } \mathrm{A \cdot m^{-1}}\frac{\mathrm{m^3}}{\mathrm{kg}}\right)$$

The SI equivalent of Equation 1.15 is

$$B = \mu_0(H + M), \tag{1.22}$$

with B in tesla and H and M in A/m. This is known as the *Sommerfeld* convention. It is equally possible to express magnetization in units of tesla, or μ_0(A/m). This is known

[6]Nicola Tesla (1856–1943), Serbian-American inventor, engineer, and scientist is largely responsible for the development of alternating current (ac) technology.

as the *Kennelly* convention, under which Equation 1.15 becomes

$$B = \mu_0 H + I \tag{1.23}$$

and I (or J) is called the *magnetic polarization*. The Sommerfeld convention is "recognized" in the SI system, and will be used henceforth in this book.

Volume susceptibility χ_V is defined as M/H, and is dimensionless. Mass susceptibility χ_m has units

$$\frac{A \cdot m^2}{kg} \cdot \frac{1}{A \cdot m^{-1}} = \frac{m^3}{kg},$$

or reciprocal density. Other susceptibilities are similarly defined.

Permeability μ is defined as B/H, and so has the units of μ_0. It is customary to use instead the *relative permeability*

$$\mu_r = \frac{\mu}{\mu_0},$$

which is dimensionless, and is numerically the same as the cgs permeability μ.

Appendix 3 gives a table of conversions between cgs and SI units.

1.9 MAGNETIZATION CURVES AND HYSTERESIS LOOPS

Both ferro- and ferrimagnetic materials differ widely in the ease with which they can be magnetized. If a small applied field suffices to produce saturation, the material is said to be *magnetically soft* (Fig. 1.14a). Saturation of some other material, which will in general have a different value of M_s, may require very large fields, as shown by curve (c). Such a material is *magnetically hard*. Sometimes the same material may be either magnetically soft or hard, depending on its physical condition: thus curve (a) might relate to a well-annealed material, and curve (b) to the heavily cold-worked state.

Figure 1.15 shows magnetization curves both in terms of B (full line from the origin in first quadrant) and M (dashed line). Although M is constant after saturation is achieved, B continues to increase with H, because H forms part of B. Equation 1.15 shows that the slope

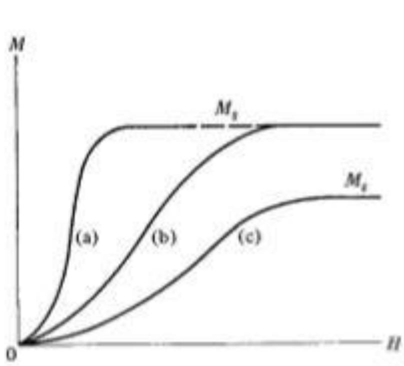

Fig. 1.14 Magnetization curves of different materials.

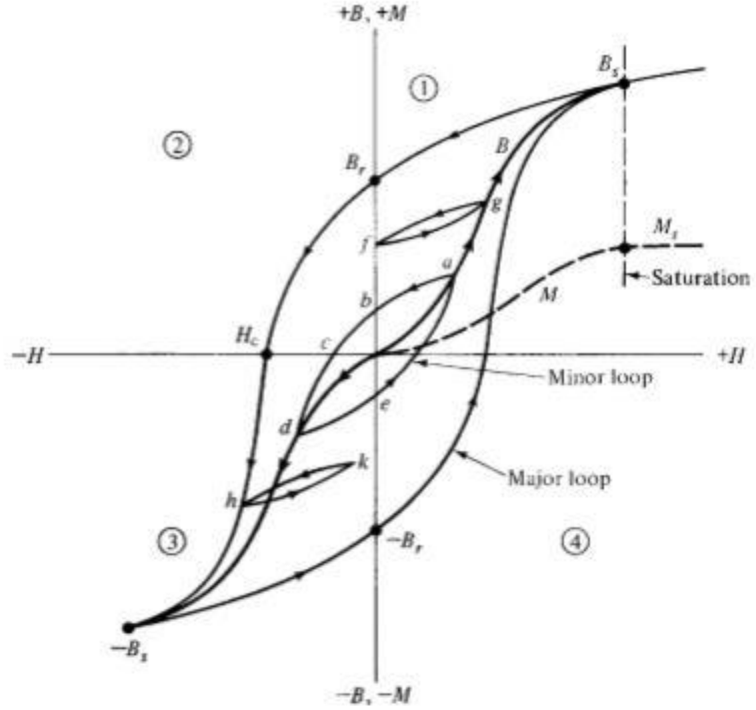

Fig. 1.15 Magnetization curves and hysteresis loops. (The height of the M curve is exaggerated relative to that of the B curve.)

dB/dH is unity beyond the point B_s, called the *saturation induction*; however, the slope of this line does not normally *appear* to be unity, because the B and H scales are usually quite different. Continued increase of H beyond saturation will cause μ(cgs) or μ_r(SI) to approach 1 as H approaches infinity. The curve of B vs H from the demagnetized state to saturation is called the *normal magnetization* or *normal induction* curve. It may be measured in two different ways, and the demagnetized state also may be achieved in two different ways, as will be noted later in this chapter. The differences are not practically significant in most cases.

Sometimes in cgs units the *intrinsic induction*, or *ferric induction*, $B_i = B - H$, is plotted as a function of H. Since $B - H = 4\pi M$, such a curve will differ from an M, H curve only by a factor of 4π applied to the ordinate. B_i measures the number of lines of magnetization/cm^2, not counting the flux lines due to the applied field.

If H is reduced to zero after saturation has been reached in the positive direction, the induction in a ring specimen will decrease from B_s to B_r, called the *retentivity* or *residual induction*. If the applied field is then reversed, by reversing the current in the magnetizing winding, the induction will decrease to zero when the negative applied field equals the *coercivity* H_c. This is the reverse field necessary to "coerce" the material back to zero induction; it is usually written as a positive quantity, the negative sign being understood. At this point, M is still positive and is given by $|H_C/4\pi|$ (cgs) or H_C (SI). The reverse field required to reduce M to zero is called the *intrinsic coercivity* H_{ci} (or sometimes ${}_iH_c$ or H_c^i). To emphasize the difference between the two coercivities, some authors write ${}_BH_c$ for the coercivity and ${}_MH_c$ for the intrinsic coercivity. The difference between H_c and H_{ci} is usually negligible for soft magnetic materials, but may be substantial for permanent magnet materials. This point will be considered further in our consideration of permanent magnet materials in Chapter 14.

If the reversed field is further increased, saturation in the reverse direction will be reached at $-B_s$. If the field is then reduced to zero and applied in the original direction, the induction will follow the curve $-B_s$, $-B_r$, $+B_s$. The loop traced out is known as the *major hysteresis loop*, when both tips represent saturation. It is symmetrical about the origin as a point of inversion, i.e., if the right-hand half of the loop is rotated 180° about the H axis, it will be the mirror image of the left-hand half. The loop quadrants are numbered 1–4 (or sometimes I–IV) counterclockwise, as shown in Fig. 1.15, since this is the order in which they are usually traversed.

If the process of initial magnetization is interrupted at some intermediate point such as a and the corresponding field is reversed and then reapplied, the induction will travel around the minor hysteresis loop *abcdea*. Here b is called the *remanence* and c the *coercive field* (or in older literature the *coercive force*). (Despite the definitions given here, the terms *remanence* and *retentivity*, and *coercive field* and *coercivity*, are often used interchangeably. In particular, the term coercive field is often loosely applied to any field, including H_c, which reduces B to zero, whether the specimen has been previously saturated or not. When "coercive field" is used without any other qualification, it is usually safe to assume that "coercivity" is actually meant.)

There are an infinite number of symmetrical minor hysteresis loops inside the major loop, and the curve produced by joining their tips gives one version of the normal induction curve. There are also an infinite number of nonsymmetrical minor loops, some of which are shown at *fg* and *hk*.

If a specimen is being cycled on a symmetrical loop, it will always be magnetized in one direction or the other when H is reduced to zero. Demagnetization is accomplished by subjecting the sample to a series of alternating fields of slowly decreasing amplitude. In this way the induction is made to traverse smaller and smaller loops until it finally arrives at the origin (Fig. 1.16). This process is known as *cyclic* or *field* demagnetization. An alternative demagnetization method is to heat the sample above its *Curie point*, at which it becomes paramagnetic, and then to cool it in the absence of a magnetic field. This is called *thermal* demagnetization. The two demagnetization methods will not in general lead to identical internal magnetic structures, but the difference is inconsequential for

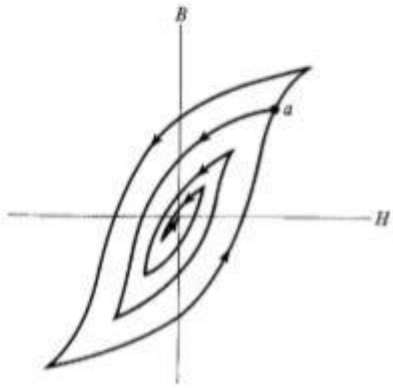

Fig. 1.16 Demagnetization by cycling with decreasing field amplitude.

most practical purposes. Some practical aspects of demagnetization are considered in the next chapter.

PROBLEMS

1.1 Magnetization M and field strength H have the same units (A/m) in SI units. Show that they have the same dimensional units (length, mass, time, current) in cgs.

1.2 A cylinder of ferromagnetic material is 6.0 cm long and 1.25 cm in diameter, and has a magnetic moment of 7.45×10^3 emu.

a. Find the magnetization of the material.

b. What current would have to be passed through a coil of 200 turns, 6.0 cm long and 1.25 cm in diameter, to produce the same magnetic moment?

c. If a more reasonable current of 1.5 ampere is passed through this coil, what is the resulting magnetic moment?

1.3 A cylinder of paramagnetic material, with the same dimensions as in the previous problem, has a volume susceptibility χ_V of 2.0×10^{-6} (SI). What is its magnetic moment and its magnetization in an applied field of 1.2 T?

1.4 A ring sample of iron has a mean diameter of 5.5 cm and a cross-sectional area of 1.2 cm^2. It is wound with a uniformly distributed winding of 250 turns. The ring is initially demagnetized, and then a current of 1.5 ampere is passed through the winding. A fluxmeter connected to a secondary winding on the ring measures a flux change of 8.25×10^{-3} weber.

a. What magnetic field is acting on the material of the ring?

b. What is the magnetization of the ring material?

c. What is the relative permeability of the ring material in this field?